Flutter of Two-Bay Flat Panels of Infinite Span at Supersonic Mach Numbers

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The two-dimensional flutter of two-bay clamped-edge and simply supported panels at supersonic Mach numbers is studied to determine the influence of structural and aerodynamic coupling. It is found that the effect of structural coupling between the panel bays is unimportant both for the "single-degree-of-freedom" flutter modes that are critical at the lower supersonic Mach numbers and for the critical coupled flutter modes that develop at higher supersonic Mach numbers. The effect of aerodynamic coupling is found to be of importance for the single-degree-of-freedom flutter modes only. Furthermore, the severity of this effect is highly dependent upon the panel boundary conditions.

Nomenclature

E = Young's modulus of the panel material
g = structural damping coefficient
h = plate thickness
k = $\omega L/2U$, reduced frequency of flutter
k_1 = $\omega_1 L/U$, stiffness parameter
L = length of panel bay
M = freestream Mach number
N = number of panel bays
$\mathbf{p}$ = aerodynamic pressure vector
$p_a(x, t)$ = aerodynamic pressure induced by deflection $w(x, t)$ acting upon upper surface of plate
t = time variable
U = freestream velocity
$w(x, t)$ = vertical deflection of panel, positive in direction of z axis
x = streamwise coordinate
$\mathbf{Y}$ = panel displacement vector
$Y(x)$ = plate mode shape
β = $(M^2 - 1)^{1/2}$, compressibility parameter
Δx = distance between collocation points
ν = Poisson's ratio of panel material
μ = $\rho_s h/\rho L$, mass parameter
ρ = freestream density
ρ_s = density of panel material
ω = flutter frequency
$\bar{\omega}$ = $2kM^2/(M^2 - 1)$, supersonic reduced frequency
ω_1 = fundamental frequency of free vibration
= $(\pi^2/L^2)[Eh^2/12\rho_s(1 - \nu^2)]^{1/2}$, simply supported panel
= $[(1.51\pi)^2/L^2][Eh^2/12\rho_s(1 - \nu^2)]^{1/2}$, clamped-edge panel

I. Introduction

THE phenomenon of panel flutter has been of technical concern since the advent of aerospace vehicles that operate at supersonic speeds. The earliest works on the subject were mainly concerned with the study of single-panel elements exposed to supersonic flows, i.e., isolated finite panels or single panels of infinite span. The investigations proved to be quite complex, and a number of difficulties arose revealing that great care was required in the analysis of the problem. Many of these difficulties were resolved after further study, and the results of the investigations were employed as a basis for the design of panel elements against flutter. (See Refs. 1–4 for discussion and bibliographies.)

In practice, however, the panel elements are not isolated but form part of the continuous skin covering of the vehicle. The various panels forming this surface may interact with one another through either aerodynamic or structural coupling. The aerodynamic coupling arises from the convection of pressure disturbances by the airstream, whereas the structural coupling arises from the continuity conditions between the adjacent panel bays. The first effect may introduce appreciable phase shifts between the aerodynamic pressures and the panel motion due to the time lag involved in the convection process; the second effect introduces new modes of vibration and flutter that are absent from the single-panel case.

The question arises, therefore, as to how realistic are the design criteria based upon the isolated panel studies. To answer this question it is necessary to study the flutter of an array of panels. However, because of the complexity of the analysis and the numerical calculations involved, it is not feasible to study such a configuration in general, and simplified versions must be chosen for analysis. Two such simplified configurations that have been studied are 1) the plate of infinite span and finite length separated into uniform rectangular panels in the spanwise direction and 2) the plate of infinite span and finite length separated into a given number N of uniform panel bays in the streamwise direction.

The earliest flutter analysis of the first configuration was carried out by Luke and St. John.[5] The panels were assumed to be simply supported or clamped at their leading and trailing edges; the streamwise edges were assumed to be simply supported. The linearized theory of inviscid supersonic flow was employed to describe the unsteady aerodynamic pressures. Under the further assumption that adjacent panels were fluttering 180° out of phase, the aerodynamic expressions appearing in the analysis were greatly simplified and were reduced to expressions similar to those of the two-dimensional case. These assumptions removed the structural coupling from the problem and fixed the nature of the aerodynamic coupling; the numerical results presented by Luke and St. John primarily demonstrate the influence of panel aspect ratio on the flutter boundaries for the particular configuration studied. (Additional numerical data will also be found in Refs. 6 and 7.)

The second configuration has been studied by Rodden,[8] Dowell,[9] and Zeijdel[6]; an extension by Rodden of his original work[8] was reported in Ref. 1; simply supported panels were treated in each case. Rodden and Zeijdel both employed the complete linearized supersonic flow theory, whereas Dowell assumed that the Mach number of the airstream was sufficiently large so that the piston theory approximation could be employed; this latter assumption removes the aerodynamic coupling effect from the problem. The analysis due to

Received December 23, 1964; revision received May 13, 1965. This work was supported by the U. S. Air Force under Contract No. AF 04(695)-269. The authors would like to express their thanks to Heather Malcom for assistance in the programing of the aerodynamic coefficients and to Mary Abell for assistance in the reduction of the numerical data. Valuable discussions with William P. Rodden are also acknowledged.

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Dowell therefore reveals the influence of structural coupling at these higher supersonic Mach numbers. The results obtained in Ref. 9 indicate that the effect is not severe, at least for $N \leq 6$, especially if realistic values of aerodynamic and structural damping are taken into account.

The alarming increases in the panel thickness ratios required to prevent flutter, with increase in the number of panel bays, that were obtained by Rodden and noted in Ref. 1 and that appeared to be at variance with the findings of Ref. 9, were later shown to be spurious.[10] This latter work, employing the complete supersonic flow theory, confirmed the findings of Ref. 9 and verified that the neglect of aerodynamic coupling at sufficiently high supersonic Mach numbers was justifiable for the simply supported panels studied.†

A certain degree of rotational restraint can be expected to be present at the supports of a typical skin panel. It is anticipated, therefore, that the stability boundaries for such panels will be intermediate to the boundaries for the limiting cases of simply supported (zero-rotational restraint) and clamped edges (infinite restraint). However, it will be noted that the structural coupling is absent for the case of clamped edges so that the stability boundaries for such panels will reduce to the corresponding single-panel boundaries if the effect of aerodynamic coupling is negligible. The combined results of Refs. 9 and 10 therefore indicate that the panel thickness requirements developed from the single-panel analysis will be realistic at sufficiently high supersonic Mach numbers if aerodynamic coupling also proves to be unimportant for the clamped-edge configurations.

The situation in the lower supersonic flows ($1 < M < 1.7$) still remains to be clarified. The analysis of Ref. 6, which concentrated upon this flow regime, included the effect of aerodynamic coupling but neglected the effects of structural coupling and structural damping. The results of this analysis reveal rather severe increases in the panel thickness ratio required to prevent flutter as the number of panel bays in the streamwise direction is increased. These increased thickness requirements were caused by extended regions of single-degree-of-freedom flutter; the significance of this trend requires verification, however, because of the neglect of both structural coupling and structural damping in the analysis. The effect of the panel boundary conditions also remains to be investigated before the significance of coupling effects at these lower supersonic Mach numbers can be assessed. Although the coupling effect is purely aerodynamic for the other limiting case of clamped-edge panels, it is not immediately apparent that the stability boundaries for such panels will exhibit trends so severe as those reported for simply supported panels in Ref. 6.

In view of the results of the previous analyses and of the critical nature of the lower supersonic Mach numbers, insofar as panel flutter is concerned, the present study is directed toward extension of the analysis in this flow regime. The second configuration was chosen for study, as it would appear to yield more pertinent information on the coupling problem. The analysis is of a conventional nature, being based upon linear plate theory and the linearized theory of inviscid supersonic flow.‡

The flutter of two-bay, clamped-edge panels at low supersonic Mach numbers is studied in some detail. The results of this analysis provide a limiting case for use with the results of simply supported panel analyses, demonstrate the influence of aerodynamic coupling at these Mach numbers, and exhibit the importance of the panel boundary conditions. The flutter of such panels at a higher supersonic Mach number is also considered, again with a view to determining the importance of aerodynamic coupling. The flutter of a two-bay simply supported panel at low supersonic Mach numbers is studied, although in less detail than the clamped-edge case. This latter study is directed to investigation of the importance of the effects of structural coupling and structural damping with a view to determining the significance of the previously reported trends. The results of these studies, together with those for simply supported panels, provide a basis for the estimation of the practical significance of coupling effects at supersonic Mach numbers.

II. Analysis

Consider a uniform flat plate of infinite span exposed to a supersonic airstream over the upper surface. The plate is assumed to be supported at $N + 1$ points, these points being equally spaced at a distance L (Fig. 1). The deformation of the panels is assumed to be describable by the Kirchhoff-Love theory of plates; assuming zero midplane stress, the vertical displacement of the panel surface is governed by an integral equation of the form

$$w(x, t) = \int_0^{NL} C(x, \xi) f(\xi, t) d\xi \tag{1}$$

where $C(x, \xi)$ is the structural influence function that gives the deflection at point x caused by a unit concentrated load located at the point ξ. The panel boundary conditions are incorporated in the influence function. The function $f(\xi, t)$ denotes the distributed loading acting upon the panel surface. This loading consists of both inertial and aerodynamic forces. Neglecting rotational inertia, one may write the function $f(\xi, t)$ as

$$f(\xi, t) = -\rho_s h(\partial^2 w/\partial t^2) + p_a(\xi, t) \tag{2}$$

where $p_a(\xi, t)$, ρ_s, and h denote the aerodynamic pressure, the density of the panel material, and the thickness of the plate, respectively.

Acoustic pressures developed on the lower surface of the panel are neglected. For the particular case of harmonic motion, the displacement, pressure, and distributed loading may be written as

$$w(x, t) = Re[Y(x)e^{i\omega t}] \tag{3a}$$

$$p_a(x, t) = Re[\bar{p}(x)e^{i\omega t}] \tag{3b}$$

$$f(x, t) = Re[\bar{f}(x)e^{i\omega t}] \tag{3c}$$

where ω denotes the frequency of the oscillations. Substitution of these expressions into Eq. (1) yields the following equation for $Y(x)$:

$$Y(x) = \int_0^{NL} C(x, \xi)[\rho_s h\omega^2 Y(\xi) + \bar{p}(\xi)]d\xi \tag{4}$$

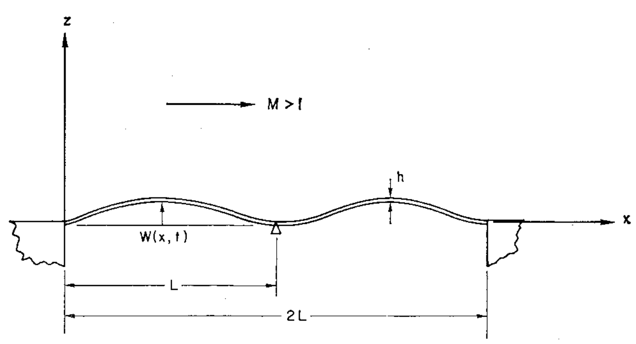

Fig. 1 Two-bay panel configuration.

† The lower Mach number limit at which aerodynamic coupling may be sensibly neglected can be expected to depend upon the number of panel bays.

‡ Objection could be raised concerning the use of the inviscid flow theory at the lower supersonic Mach numbers because of the apparent importance of the fluid boundary layer.[11] However, as noted in the preceding paragraphs, the multiple-bay panel flutter problem has not yet been fully resolved even in the inviscid case. Furthermore, it is anticipated that the inviscid analysis will be of sufficient accuracy to establish correctly the existence and importance of specific effects and trends.

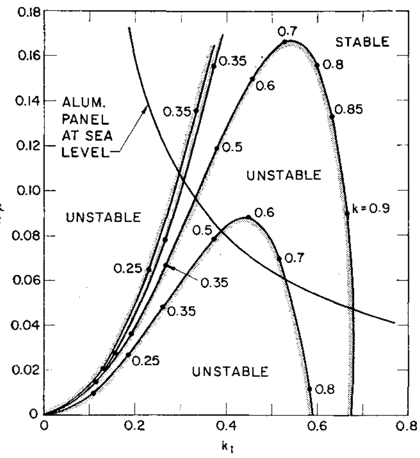

Fig. 2 Stability boundaries for two-bay clamped-edge panel, $M = (2)^{1/2}$, $g = 0$.

Equation (4) is now collocated at a given number of points x_i over the plate span; the collocation points are assumed to be equally spaced in the individual panel bays. Employing m collocation points, one obtains the following set of equations:

$$Y(x_i) = \int_0^{NL} C(x_i, \xi)[\rho_s h\omega^2 Y(\xi) + \bar{p}(\xi)]d\xi$$
$$(i = 1, 2, \ldots m) \tag{5}$$

The terms appearing on the right-hand side of the preceding equations are now approximately integrated through application of the trapezoidal rule. The following matrix equation is then obtained:

$$\mathbf{Y} = [C](\rho_s h\omega^2 \mathbf{Y} + \mathbf{p})\Delta x \tag{6}$$

where Δx denotes the spacing of the collocation points in the individual panel bays. The vector $\mathbf{Y}$ comprises the set of panel displacements at the collocation points, and the vector $\mathbf{p}$ comprises the corresponding set of aerodynamic pressures. The matrix $[C]$ is the flexibility matrix, the elements of this matrix being defined as follows:

$$c_{ij} = C(x_i, \xi_j) \tag{7}$$

The aerodynamic pressure vector $\mathbf{p}$ is now related to the displacement vector $\mathbf{Y}$ by the equation

$$\mathbf{p} = (\rho\omega^2 L^2/4\Delta x)[C_h]\mathbf{Y} \tag{8}$$

where ρ denotes the freestream density. The matrix $[C_h]$ is the matrix of aerodynamic influence coefficients. The particular form of the elements of this matrix is governed by the aerodynamic theory employed to describe the unsteady pressures and by the numerical techniques employed to reduce the pressure expressions to matrix form.

The linearized theory of inviscid supersonic flow will be employed herein; following this theory,[12] one may write the complex aerodynamic pressure amplitude as

$$\bar{p}(x) = -\frac{\rho U^2}{\beta}\left[\frac{dY}{dx} + \frac{2ik}{L}\frac{(M^2-2)}{\beta^2}Y(x) + \frac{4k^2}{\beta^4 L^2}\int_0^x Y(\xi)E(x-\xi)d\xi\right] \tag{9}$$

where

$$E(x) = \exp(-i\bar{\omega}x/L)F(x) \tag{10a}$$

$$F(x) = -[(M^2+2)/2]J_0(\bar{\omega}x/ML) + (M^2/2)J_2(\bar{\omega}x/ML) + 2iMJ_1(\bar{\omega}x/ML) \tag{10b}$$

and where U denotes the freestream velocity, and M denotes the freestream Mach number. The functions $J_0(x)$, $J_1(x)$, and $J_2(x)$ are Bessel functions of the first kind of orders 0, 1, and 2, respectively. The parameters β, k, and $\bar{\omega}$ are defined as follows:

$$\beta = (M^2-1)^{1/2} \qquad k = \omega L/2U$$
$$\bar{\omega} = 2kM^2/(M^2-1) \tag{11}$$

The first term in Eq. (9) represents the quasi-static theory; the first and second terms comprise the quasi-steady theory. The elements of the matrix $[C_h]$ are now obtained by application of numerical differentiation and integration formulas to Eq. (9); the details of this process will be found in Ref. 13.

Equation (8) is now substituted into Eq. (6). Introducing the parameters

$$1/\mu = \rho L/\rho_s h \qquad k_1 = \omega_1 L/U \tag{12}$$

where ω_1 denotes the lowest natural frequency of the system in vacuo, one may express the matrix equation of motion in the form

$$\lambda\mathbf{Y} = [G]\mathbf{Y} \tag{13}$$

where the matrix $[G]$ depends upon the parameters k, M, and $1/\mu$. The eigenvalue λ is defined by

$$\lambda = (k_1/k)^2 \tag{14}$$

The effect of damping is introduced in the conventional manner by means of the structural damping coefficient g. The stiffness parameter k_1^2 is simply replaced by $k_1^2(1 + ig)$, where the extra term represents a damping force proportional to the amplitude of oscillation. The eigenvalue for this case becomes

$$\lambda = (k_1/k)^2(1 + ig) \tag{15}$$

The eigenvalues of (13) are determined through application of the power method of iteration; the details of the particular computational procedure employed in this study are described in Appendix A of Ref. 14.

III. Numerical Results and Discussion

The numerical procedure consisted of specifying a particular Mach number, calculating the eigenvalues corresponding to given values of k and $1/\mu$, and then determining the parameters k_1 and g from the eigenvalues. The first four eigenvalues of the system were calculated for each case; higher eigenvalues were not determined because it appears doubtful that the corresponding flutter modes would be of practical significance. The calculations were performed with Δx

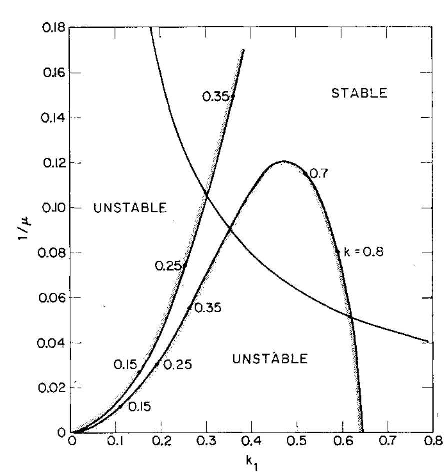

Fig. 3 Stability boundaries for single-bay clamped-edge panel, $M = (2)^{1/2}$, $g = 0$.

equal to 0.1 L; the two-bay panels are therefore approximated by an 18-degrees-of-freedom system. The adequacy of this number of collocation points for the two-bay problem has been noted previously in Ref. 10.

Flutter boundaries were obtained for two-bay clamped-edge panels at Mach numbers of 1.2, 1.3, 1.35, $(2)^{1/2}$, 1.45, 1.47, 1.52, and 2.0; flutter boundaries for a two-bay simply supported panel were determined for Mach numbers of 1.3 and 1.52. These stability boundaries are presented extensively in Ref. 15 in the form of $1/\mu$ vs k_1 curves for given Mach number and structural damping; selected curves from this reference are shown in Figs. 2–6 of the current paper. The flutter boundaries may also be expressed in terms of the minimum panel thickness ratio required to prevent flutter as a function of Mach number for a given panel material and structural damping by using the result that

$$k_1/\mu = \text{const}\{(\rho^2/\rho_s{}^3)[E/(1-\nu^2)U^2]\}^{1/2}$$

Each panel material and flight condition, therefore, specifies a hyperbola whose intersections with the flutter boundaries in the $(k_1, 1/\mu)$ plane determine the critical panel thickness ratios. The results of the calculations will now be discussed in some detail.

Clamped-Edge Panels

Comparison of the results for the two-bay clamped-edge panels with those for the single-bay panel revealed that the general behavior of the stability boundaries in the $(k_1, 1/\mu)$ plane is similar in both cases. A typical set of stability curves for a two-bay panel is shown in Fig. 2 where the results for a Mach number of $(2)^{1/2}$ are presented for the case of zero structural damping; these boundaries may be compared with those shown in Fig. 3 where the corresponding results for a single-bay panel are displayed. Hyperbolas defined by the physical constants corresponding to aluminum panels at sea-level flight conditions are also shown in these figures.

This behavior is not altogether surprising. The clamped-edge boundary conditions preclude structural coupling; hence, no additional modes of vibration are introduced into the problem. The free vibration frequencies of the multiple-bay clamped-edge panel are therefore identical to those of the corresponding single-bay panel. However, the frequencies are now degenerate; if a multiple-bay panel has N bays, then the frequencies will have an N-fold degeneracy. The effect of the aerodynamic coupling is to separate the modes corresponding to a particular vibration frequency. However, if the coupling is relatively weak, the modes of the multiple-bay system can be expected to behave in a manner similar to that of the corresponding modes of the single-bay case. The results of the flutter calculations indicate therefore that such coupling is indeed weak for the clamped-edge configuration studied.

The most important difference between the results obtained for the two-bay case and those of the single-bay panel is the extension of the regions of instability (compare Figs. 2 and 3, for example). The significance of these extended regions of instability is best illustrated by examining typical stability boundaries in the $(h/L, M)$ plane. Such boundaries are shown in Figs. 7 and 8, where flutter results for aluminum panels at sea-level flight conditions are presented for values

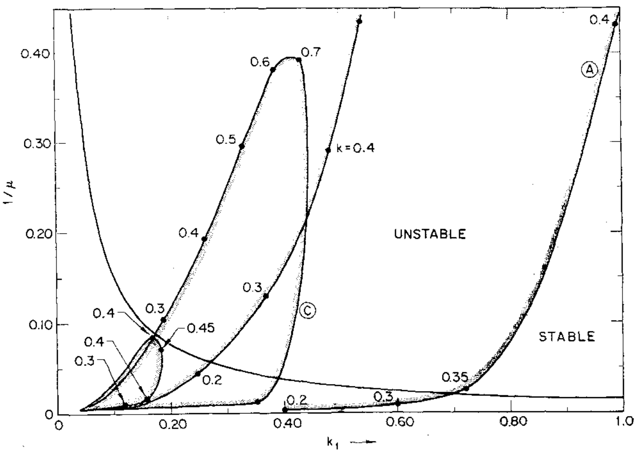

Fig. 5 Stability boundaries for two-bay simply supported panel, M = 1.3, g = 0.01.

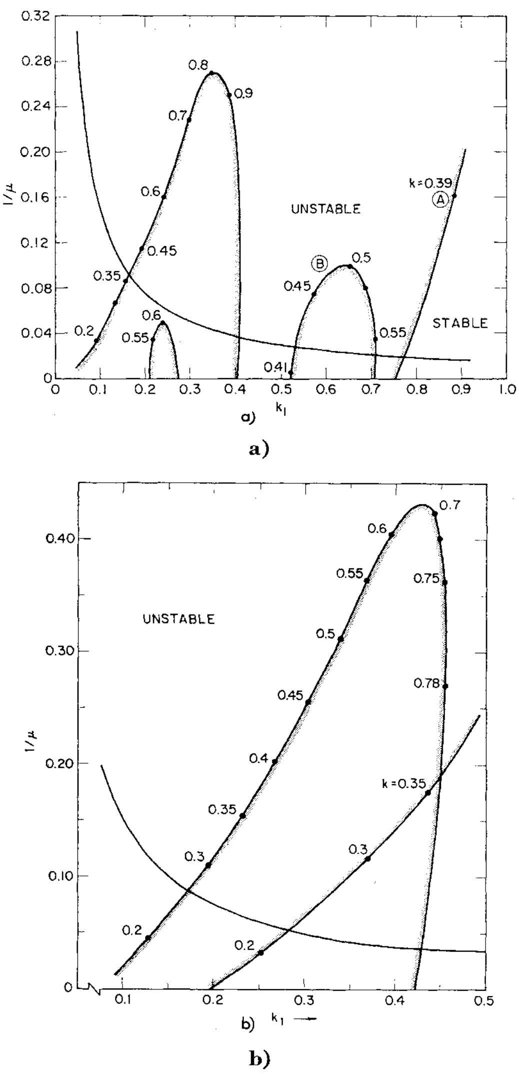

Fig. 4 Stability boundaries for two-bay simply supported panel, M = 1.3, g = 0.

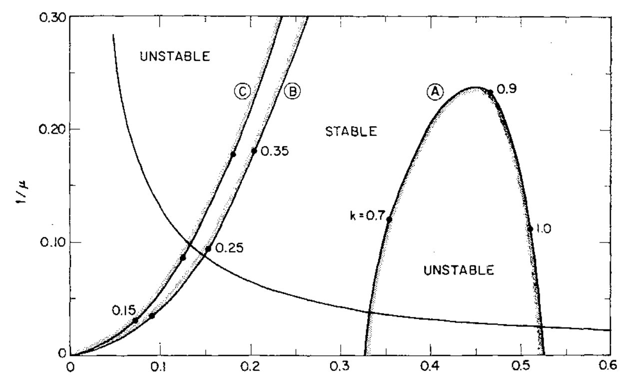

Fig. 6 Stability boundaries for two-bay simply supported panel, M = 1.52, g = 0.

of g equal to zero and 0.01. These curves were evaluated using the following physical constants:

$$E = 10.5 \times 10^6 \text{ lb/in.}^2 \qquad \rho_s = 5.374 \text{ slug/ft}^3$$
$$\rho = 0.002378 \text{ slug/ft}^3 \qquad \nu = 0.318 \qquad a = 1117 \text{ fps}$$

The corresponding boundaries for the single-bay panel are shown in Figs. 9 and 10. The estimated frequency ratios ω/ω_1 at flutter are also noted on these figures. Considering first the case of zero structural damping (Fig. 7), one will note that the critical flutter mode changes from a single-degree-of-freedom first-mode flutter§ at Mach numbers between 1.2 and 1.35 to a critical single-degree-of-freedom second-mode flutter at Mach numbers up to 1.46 which in turn is replaced by a critical coupled mode flutter. This behavior is again similar to that occurring in the single-bay case. The coupled flutter modes yield the critical stability boundaries at the higher supersonic Mach numbers; however, in the Mach number range being studied, $1.2 \leq M \leq 2$, the maximum required panel thickness ratio for fixed flight conditions is determined by the single-degree-of-freedom first-mode boundary.

Comparison of the single-bay and two-bay panel results reveals that the first-mode thickness ratio requirements are increased by approximately 4 to 11% in the Mach number range between 1.2 and $(2)^{1/2}$, the maximum required thickness ratio being increased by about 5%. This latter result compares with an increase of approximately 35% reported by Zeijdel[6] for a two-bay simply supported panel. These increases arise from changes in the 'single-degree-of-freedom' stability boundaries and are both a result of the effect of aerodynamic coupling alone (structural coupling was neglected in Ref. 6), the substantial difference between results for the clamped-edge and the simply supported panel therefore indicating that the effect of aerodynamic coupling upon the single-degree-of-freedom flutter modes is highly dependent upon the panel boundary conditions. In contrast to this behavior, the coupled mode flutter boundaries that develop for Mach numbers above $(2)^{1/2}$ are virtually identical to the corresponding boundary in the single-bay case, the close agreement between these boundaries demonstrating that aerodynamic coupling has a negligible effect upon this type of flutter motion.

The effect of the specified structural damping is to reduce the regions of instability at the lower supersonic Mach numbers (Fig. 8), the maximum required thickness ratio in the range $1.2 \leq M \leq 2$ being reduced by slightly over 8% from the value obtained in the case of zero structural damping.

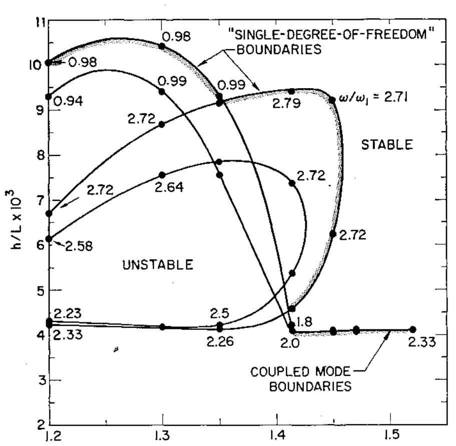

Fig. 7 Thickness ratio vs Mach number curves for two-bay clamped-edge aluminum panel at sea level, $g = 0$.

§ Flutter modes characterized by displacements of essentially constant phase and by frequencies similar to particular free vibration frequencies.

The effect is most pronounced for the second-mode stability curves, which are now critical only over a very limited range of Mach number. The coupled mode stability boundaries are only slightly affected by the given structural damping.

Simply Supported Panels

The stability boundaries for the simply supported panels are more complicated than those of the clamped-edge case because of the presence of the additional modes introduced by the structural coupling. For the case of the two-bay panel, the distribution of free vibration frequencies, as measured by the ratios of the vibration frequencies ω_n to the lowest vibration frequency ω_1, is 1.0, 1.562, 4.0, 5.061, 9.0, . . . , whereas the distribution for the single-bay case is 1.0, 4.0, 9.0, ... However, examination of the stability boundaries calculated at Mach numbers 1.3 and 1.52 reveals that these modes are not critical in themselves. The stability boundaries at a Mach number of 1.3, presented in Figs. 4a and 4b for the case of zero structural damping, show that the critical flutter mode is the single-degree-of-freedom first-mode flutter, which is also present when structural coupling is neglected (see curve A in Fig. 4a). The critical thickness ratio for an aluminum panel at sea-level flight conditions corresponding to this mode is 0.0224; the same critical value was obtained by Zeijdel in Ref. 6. The increase of the required thickness ratio over the corresponding result for the single-bay case is approximately 26%. The next flutter boundary derives from one of the modes introduced by the structural coupling (see curve B in Fig. 4a). This mode is also a single-degree-of-freedom flutter mode with a frequency ratio of approximately 1.56. The critical thickness ratio corresponding to this mode is slightly less than that associated with the first-mode boundary.

The introduction of structural damping, to the extent of g equal to 0.01, does not change the nature of the critical flutter mode but does reduce the maximum required panel thickness ratio by about 11%. The second flutter mode, which had a frequency ratio of 1.56, is completely eliminated by the presence of the given amount of structural damping. The next flutter mode (curve C in Fig. 5) is now associated with a higher frequency motion and has a critical thickness ratio of 0.0116.

The critical flutter mode at Mach number 1.52, for zero structural damping, is again a single-degree-of-freedom mode (see curve A in Fig. 6), its frequency ratio being approximately 4.0. The corresponding critical thickness ratio for an aluminum panel at sea level is approximately 0.0177. As in the previous case, the value obtained by Zeijdel is in close agreement. The remaining two stability boundaries (see curves B and C in Fig. 6) are associated with coupled

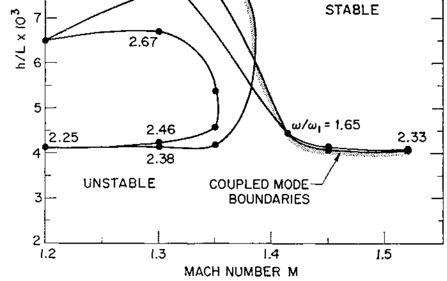

Fig. 8 Thickness ratio vs Mach number curves for two-bay clamped-edge aluminum panel at sea level, $g = 0.01$.

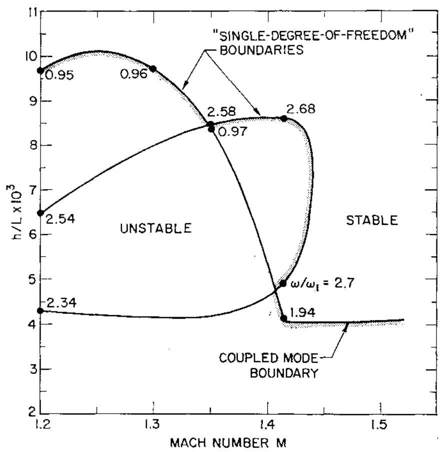

Fig. 9 Thickness ratio vs Mach number curves for single-bay clamped-edge aluminum panel at sea level, $g = 0$.

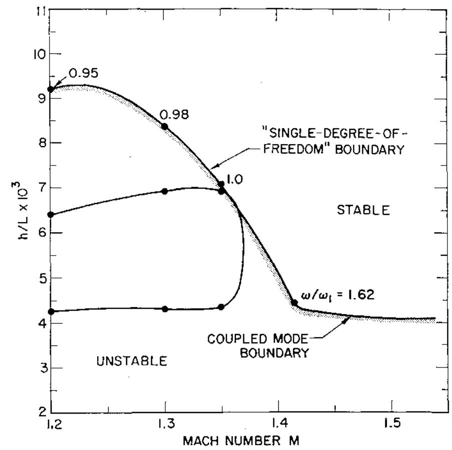

Fig. 10 Thickness ratio vs Mach number curves for single-bay clamped-edge aluminum panel at sea level, $g = 0.01$.

flutter modes, the respective frequency ratios being approximately 3.2 and 3.95.

The addition of structural damping, in the amount of g equal to 0.01, completely eliminates the critical single-degree-of-freedom flutter mode but has little effect on the coupled modes. The required panel thickness ratios are now generated by these coupled modes. The maximum required thickness ratio for the case is now 0.0049. This value agrees very closely with the single-panel requirements of approximately 0.005, thereby demonstrating the negligible effect of both aerodynamic and structural coupling on the critical coupled mode flutter boundary.

IV. Concluding Remarks

Reviewing the results of the present analysis together with the results of Refs. 6, 9, and 10, one can draw the following conclusions for the two-bay configuration treated herein.

1) The effect of structural coupling between the panel bays is unimportant both for the single-degree-of-freedom flutter modes that are critical at the lower supersonic Mach numbers and for the critical coupled flutter modes that develop at higher supersonic Mach numbers.

2) The effect of aerodynamic coupling is of importance only for the critical single-degree-of-freedom flutter modes that appear at the lower supersonic Mach numbers; however, the severity of the effect is highly dependent upon the panel boundary conditions. Furthermore, the presence of realistic amounts of structural damping should be included in the calculation of flutter boundaries for these modes.

It is seen, therefore, that the single-bay panel results are realistic for the higher supersonic Mach numbers. The presence of adjacent panels appears to be of some importance at the lower supersonic Mach numbers because of the influence of aerodynamic coupling upon the single-degree-of-freedom flutter modes that are critical in this regime. In connection with this effect, however, it should be noted that previous investigation[11] of single-bay panels has revealed that the stability boundaries calculated for such modes with the use of the inviscid supersonic flow theory are quite conservative compared to experimental data. The stability boundaries obtained for the single-degree-of-freedom flutter modes in the case of the two-bay panels are therefore also expected to be somewhat conservative; however, the percentage changes in the required panel thickness ratios may serve as a useful guide for design purposes.

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